A quick guide to the influence of sampling frequency on the estimation of the young water fraction from isotopic measurements in streamwater

Final report of working group 3 of the COST ACTION WATSON

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On the number of samples needed to obtain sound estimates of a catchment's young water fraction and transit time distribution

1. Introduction

The "young water fraction" (Fyw) has been introduced by Kirchner (2016) as a robust alternative to the mean transit time obtained by fitted a transit time distribution to tracer data. The young water fraction is the fraction of streamflow with transit time through the hydrological system of less than 2 to 3 months and is estimated from the amplitude damping between the input (usually precipitation) and output signals. In this document, we present how variations in stream discharge seem to influence the estimation of the young water fraction and explore the relationship between the number of samples and the estimation error.

2. Method

It is well known that stream discharges follow log-normal distributions. In these distributions, the mode is smaller than the median and the median is smaller than de mean, and these differences increase with the flow variance. The fact that the samples with higher frequency are smaller than the mean value infers that i) a series with small number of samples tends to deliver a negatively biased mean value, and ii) diverse sampling series with small number of samples tend to deliver different mean values that converge with increasing number of samples.

Increasing streamflow discharge is usually associated with higher young water fraction (Kirchner 2016) and younger mean transit times (MTT, Harman, 2015). Therefore, we can expect that the statistical properties of discharge are usually propagated to the age of water. Indeed, evidence shows that: i) a negative bias of Fyw and longer MTT with decreasing sampling frequency was first shown by Stockinger et al. (2016), and ii) diverging Fyw estimates from different sampling series were obtained by Stockinger et al. (2019).

A virtual experiment can help to analyse how stream discharge characteristics and Fyw discharge sensitivity influence the number of samples needed to obtain unbiased and low uncertainty Fyw estimates, as a surrogate of the quality of the MTT that may be obtained from these data: For every one of 19 catchments where the Fyw characteristics are known (Von Freyberg et al., 2018; Gallart et al., 2020; Xia et al., 2024), we generated ten series of 10.000 random uncorrelated log-normally distributed samples of discharge. Then, we simulated the Fyw corresponding to every one of the samples using the equation (1) proposed by Gallart et al. (2020), where F_0 (-) is the F_{yw} for virtual Q=0 and S_d (unit of Q^{-1}) is the discharge sensitivity metric.

$$F_{\mathrm{yw}}(Q) = 1 - (1 - F_0)\exp(-Q \cdot S_{\mathrm{d}}) \quad (1)$$

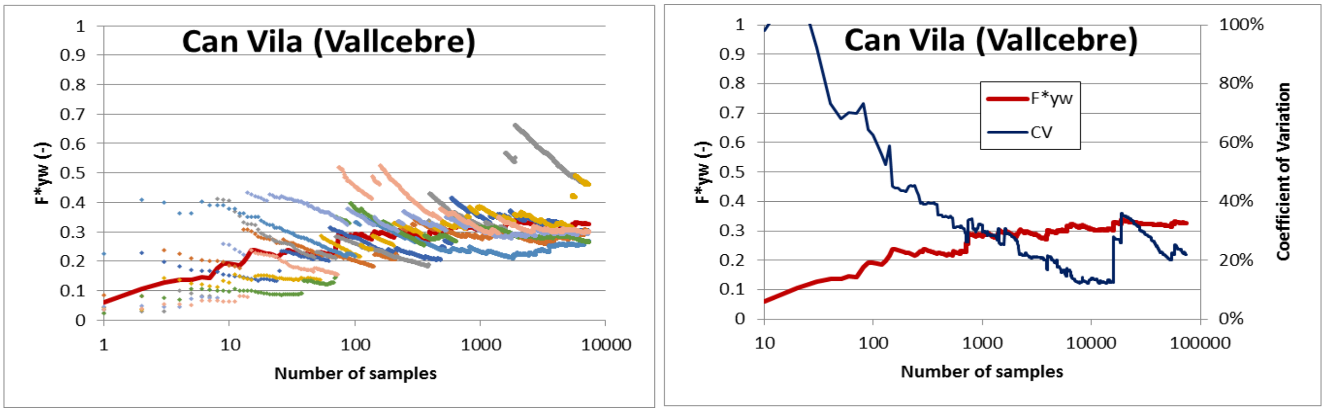

Figure 1: Results for the more varying catchment. Left: ten series of virtual random samples and their mean (red line). Right: mean and coefficient of variation obtained with all the samples. Note the strong negative bias of the mean observed for less than 1,000 samples and the large number of samples needed to obtain small values of the coefficient of variation.

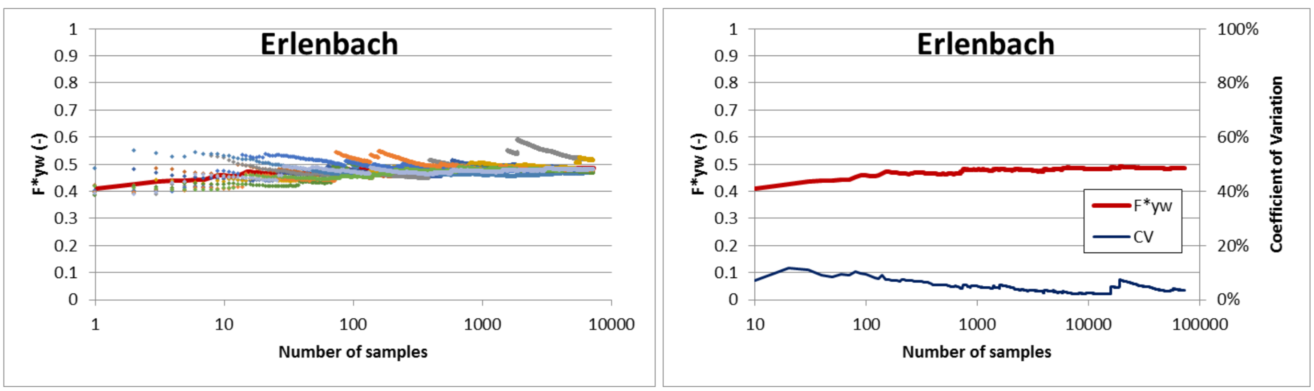

Figure 2: Results for one of the least varying catchments. Left: ten series of virtual random F*yw samples and their mean (red line). Right: mean and coefficient of variation obtained with the increasing number of F*yw samples. Both the initial negative bias and the progressive decrease of the coefficient of variation are evident although weaker.

The number of samples needed to obtain a coefficient of variation of F*yw smaller than 10% is used a benchmark to analyse the role of the diverse factors. The results obtained using data from 19 catchments shows that all the catchments needed more than 100 samples to reach the 10% CV benchmark and that the catchments with smaller mean discharge significantly needed more samples. As it could be anticipated, catchments with larger Fyw discharge sensitivity (Sd) and smaller minimal Fyw (F_0) needed more samples.

Table 1: Characteristics of the studied catchments, with an estimate of the number of samples needed to obtain a coefficient of variation of F*yw smaller than 10%.

Catchment	mean Q (mm day^{-1})	Sdev Q (mm day^{-1})	CVQ (%)	F_0 (-)	Sd (day·mm^{-1})	F*yw (-)	n. samples (-)
Erlenbach	4.55	7.73	170%	0.382	0.012	0.49	100
Langeten	1.77	1.09	62%	-0.043	0.070	0.12	700
Biber	3.15	4.71	149%	0.170	0.058	0.46	710
Can Vila	0.98	2.98	304%	0.020	0.062	0.30	7540

Lümpenenbach	4.98	7.05	141%	0.246	0.016	0.39	380
Riale2	2.85	7.47	262%	0.13	0.120	0.67	740
Aabach	1.800	2.28	126%	0.122	0.059	0.22	710
Sense	1.777	1.81	102%	-0.043	0.077	0.29	2140
Dischmabach	3.262	2.79	86%	0.019	0.017	0.10	710
Alp	4.141	5.75	139%	0.263	0.013	0.35	150
Emme	2.908	4.13	142%	0.260	0.019	0.32	170
Ergolz	1.295	1.53	118%	-0.009	0.112	0.14	1950
Allenbach	3.584	3.61	101%	0.104	0.010	0.11	290
Guerbe	3.289	2.97	90%	0.018	0.050	0.18	700
Murg	2.059	2.00	97%	0.045	0.028	0.13	710
Ova	2.387	2.12	89%	0.033	0.034	0.14	710
Rietzholzbach	2.859	4.02	141%	0.055	0.020	0.20	2730
Schaechen	3.944	3.41	86%	0.066	0.018	0.18	550
Vogelbach	3.846	6.26	163%	0.214	0.012	0.31	390

Table 2: correlation coefficients between the data shown in Table 1. Values shown in **bold** typeface are significant at the 1% level and values shown <u>underlined</u> are significant at the 5% level.

	mean Q	Sdev Q	CVQ	F0	Sd	F*yw	n. samples	log(n.s.)
mean Q	1							
Sdev Q	**0.7526**	1						
CVQ	-0.1018	<u>0.5107</u>	1					
F0	**0.7178**	**0.8359**	0.2933	1				
Sd	**-0.6530**	-0.2520	0.2884	<u>-0.4708</u>	1			
F*yw	0.3292	**0.8084**	**0.6918**	**0.6158**	0.2395	1		
n. samples	**-0.5900**	-0.2640	**0.5897**	-0.4062	0.2994	-0.0430	1	
log(n.s.)	**-0.7355**	<u>-0.5215</u>	0.2652	**-0.7609**	<u>0.5294</u>	-0.2305	**0.8196**	1

3. Conclusions

From these relationships we can conclude that the number of random samples needed to obtain a coefficient of variation of F*yw smaller than 10% are driven by mean discharge and the initial Fyw for zero discharge (S_0), both correlated, whereas the coefficient of variation of discharge and the discharge sensitivity of Fyw (Sd) are secondary drivers.

Finally, Figure 3 shows that more than 100 and typically about 1,000 random samples are needed to obtain reasonable F*yw estimates and, comparably, Transit Time Distribution estimates for most of the studied catchments. If the samples are taken not randomly but following 'smart' strategies to improve the representativeness of high discharges, the adequate number of samples might be largely reduced.

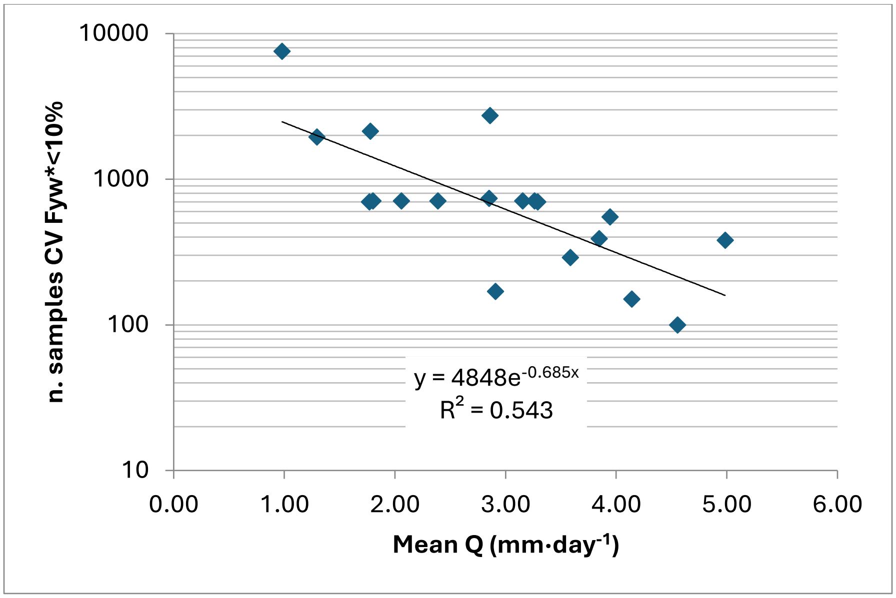

Figure 3: Number of random samples needed to obtain coefficients o variation of F*yw smaller than 10% in relationship with mean discharges. In spite of the large scatter, the relationship is significant at the 1°/₀₀ level.

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